Sample Questions for PRMO 2017

1. Two positive integers a and b are such that $a+b=\frac{a}{b}+\frac{b}{a}$. What is the value of a^2+b^2? [Ans: 02]

2. The equations $x^2-4x+k=0$ and $x^2+kx-4=0$, where k is a real number, have exactly one common root. What is the value of k? [Ans: 03]

3. Let $P(x)$ be a non-zero polynomial with integer coefficients. If $P(n)$ is divisible by n for each positive integer n, what is the value of $P(0)$? [Ans: 00]

4. A natural number k is such that $k^2 < 2014 < (k+1)^2$. What is the largest prime factor of k? [Ans: 11]

5. How many two-digit positive integers N have the property that the sum of N and the number obtained reversing the order of the digits of N is a perfect square? [Ans: 08]

6. What is the greatest possible perimeter of a right-angled triangle with integer side lengths if one of the sides has length 12? [Ans: 84]

7. In rectangle $ABCD$, $AB=8$ and $BC=20$. Let$P$ be a point on AD such that $\angle BPC=90°$. If r_1, r_2, r_3 are the radii of the incircles of triangles APB, BPC and CPD, what is the value of $r_1+r_2+r_3$? [Ans: 08]

8. Let n be the largest integer that is the product of exactly 3 distinct prime numbers, x, y and $10x+y$, where x and y are digits. What is the sum of the digits of n? [Ans: 12]

9. A subset B of the set of first 100 positive integers has the property that no two elements of B sum to 125. What is the maximum possible number of elements in B? [Ans: 62]

10. The circle ω touches the circle Ω internally at P. The centre O of Ω is outside ω. Let XY be a diameter of Ω which is also tangent to ω. Assume $PY > PX$. Let PY intersect ω at Z. If $YZ=2PZ$,what is the magnitude of $\angle PYX$ in degrees? [Ans: 15]